LNCT GROUP OF COLLEGES

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Unit: IV

Topic: Frequency Domain Analysis

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## FREQUENCY DOMAIN ANALYSIS

"The steady state response of a system to a purely sinusoidal input is defined as frequency response of a system."

In such method frequency of the input signal is to be varied over a certain range and the resulting response of the system is to be studied, which is known as frequency response.

→ * When linear time invariant system is subjected to a sinusoidal input, it will produce sinusoidal o/p at steady state having same frequency, as that of the input. But the amplitude i.e. magnitude and phase angle of o/p is different from that of I/p.

→ Frequency domain transfer function can be obtained by substituting $s = j\omega$ in the transfer function $G(s)$ of the system.

$$G(j\omega) = G(s)\Big|_{s=j\omega} = \text{freq. domain transfer function.}$$

→ So to get freq. response means to sketch the variation in magnitude and phase angle of $G(j\omega)$, when $\omega$ is varied from 0 to $\infty$

$$G(j\omega) = M \angle \phi$$

where $M$ = Magnitude → $f(\omega)$

$\phi$ = Phase Angle → $f(\omega)$

where $\omega$ = Input frequency.

* $R(s) \rightarrow \boxed{G(s)} \rightarrow C(s)$

$G(s) = G(j\omega)$

$G(j\omega) = |G(j\omega)| \angle G(j\omega)$

$\sin \omega_0 t \rightarrow \boxed{G(j\omega)} \rightarrow |G(j\omega)| \sin(\omega_0 t + \angle G(j\omega))$

$\sin t \rightarrow \boxed{\frac{1}{s+1}} \rightarrow$ o/p = ?

$G(s) = 1/s+1$

$G(j\omega) = 1/j\omega+1$

$|G(j\omega)| = 1/\sqrt{1+\omega^2} = 1/\sqrt{2}$

$\angle G(j\omega) = -\tan^{-1}\omega = -45^\circ$

$O/P = |G(j\omega)| \sin(\omega_0 t + \angle G(j\omega))$

$= \frac{1}{\sqrt{2}} \sin(t - 45^\circ)$

→ Most commonly used frequency domain methods are –

a) Bode Plot b) Polar plot 3) Nyquist plot.

## BODE PLOT

→ When $\omega$ is varied b/w 0 to $\infty$, there is wide range of variations in M & $\phi$ and it becomes difficult to accomodate all the variations with linear scale. Hence logarithmic values are plotted against logarithmic frequencies.

$\left(\frac{P_0}{P_i}\right)_{dB} = 10\log_{10}\left(\frac{P_0}{P_i}\right)$ Now $P = V^2/R$ and $I^2R$ If $R=1$, $P = V^2$

$\left(\frac{P_0}{P_i}\right)_{dB} = 10\log_{10}\left(\frac{V_0}{V_i}\right)^2 = 20\log_{10}\left(\frac{V_0}{V_i}\right) = 20\log_{10}$ (Transfer function)

* Decibel express a power ratio, not an amount
* How many times more (+dB) or less (−ve dB) but not how much in absolute terms.
* dB are logarithmic not linear

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- Bode plot consists of two plots which are-

a) Magnitude expressed in logarithmic values against logarithmic values of frequency c/d Magnitude Plot.

b) Phase angle in degrees against logarithmic values of freq. c/d phase plot

Standard form of Open loop T.F. $G(j\omega)H(j\omega)$ -

Consider $$G(s)H(s) = \frac{K' s^z (s+z_1)(s+z_2)\cdots}{s^p (s+p_1)(s+p_2)\cdots}$$

* Either $s^z$ or $s^p$ will be present at a time and not both. But this form has to be changed for sketching the Bode Plot.

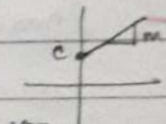
- Rewriting $G(s)H(s)$ in the time constant form.

$$G(s)H(s) = \frac{s^z K (1+sT_1)(1+sT_2)\cdots}{s^p (1+sT_a)(1+sT_b)\cdots}$$

where $K = \frac{z_1 \times z_2 \cdots}{p_1 \times p_2 \cdots} \times K'$

* for magnitude plot the eqn of straight line is used

$|G(j\omega)|_{dB} = m\log_{10}\omega + c$




when $\omega = 1$, then

$|G(j1)| = m \cdot \log_{10}(1) + c$

$|G(j1)| = c$

- The standard time constant form is denoted as,

$$G(s)H(s) = \frac{K(1+sT_1)(1+sT_2)\cdots}{s^p(1+sT_a)(1+sT_b)\cdots} \quad \text{— (A)}$$

· K = Resultant system gain and P = Type of the system

$T_1, T_2, T_a, T_b, \cdots$ = Time constants of different poles and zeros.

- Each of the factors involved in $G(s)H(s)$ will contribute to magnitude and phase angle variations of $G(j\omega)H(j\omega)$ in freq. domain.

- Frequency domain transfer function can be obtained by substituting $s = j\omega$ in eqn (A)

$$G(j\omega)H(j\omega) = \frac{K(1+j\omega T_1)(1+j\omega T_2)\cdots}{(j\omega)^p(1+j\omega T_a)(1+j\omega T_b)\cdots} \quad \text{— (B)}$$

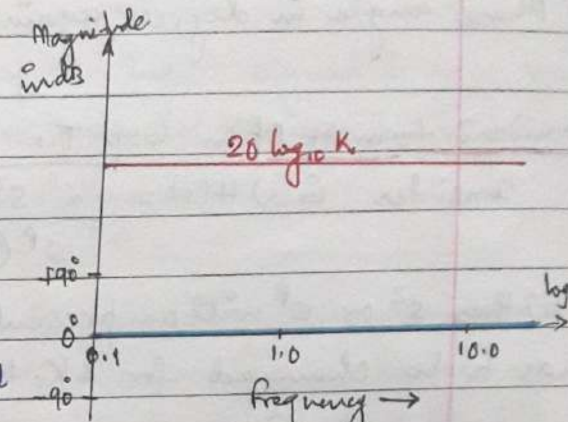
Equation (B) shows some basic factors which may appear-

1) Constant Gain factor K
2) Poles or zeros at the origin $(j\omega)^{\pm p}$

3) Simple poles and zeros $(1+j\omega T)^{\pm 1}$

4) Quadratic pole or zero $\left(1+\frac{2\xi}{\omega_n}s+\frac{s^2}{\omega_n^2}\right)$

**Factor 1: System Gain 'K'**




1) $G(s)\,H(s) = K$

i.e. $G(j\omega)\,H(j\omega) = K + j0$

$|G(j\omega)\,H(j\omega)| = \sqrt{K^2+0} = K$

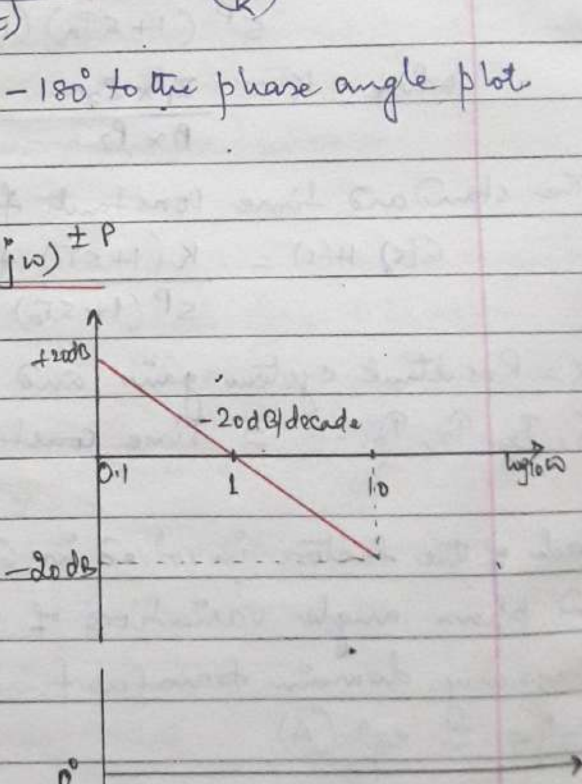
Its dB value $= 20\log_{10}K$ dB.

- As K is constant hence $20\log_{10}K$ is always a constant though $\omega$ is varied from 0 to $\infty$
- Its magnitude plot will be straight line parallel to x-axis.
- Phase angle $\phi = \tan^{-1}\frac{(\text{Imaginary part})}{(\text{Real Part})} = \tan^{-1}\left(\frac{0}{K}\right) = 0°$
- If 'K' is −ve, then it contributes −180° to the phase angle plot independent of frequency.

**Factor 2:- Poles or Zeros at the origin $(j\omega)^{\pm P}$**

2) $G(s) = 1/s$

$G(j\omega) = \frac{1}{0+j\omega}$




$|G(j\omega)| = \frac{1}{\sqrt{0^2+\omega^2}} = \frac{1}{\omega}$

$|G(j\omega)|_{dB} = 20\log_{10}(1/\omega)$

$= 20\log_{10}(1) - 20\log_{10}\omega$

$|G(j\omega)|_{dB} = \underbrace{-20}_{m}\,\underbrace{\log_{10}\omega}_{x}$

$\angle G(j\omega) = -\tan^{-1}\left(\frac{\omega}{0}\right) = -\tan^{-1}(\infty) = -90°$

Now $|G(j0.1)|_{dB} = +20\,dB$ ; $\angle G(j\omega) = -90°$

$|G(j1)|_{dB} = 0\,dB$

$|G(j10)|_{dB} = -20\,dB$

3) $G(s) = 1/s^n$

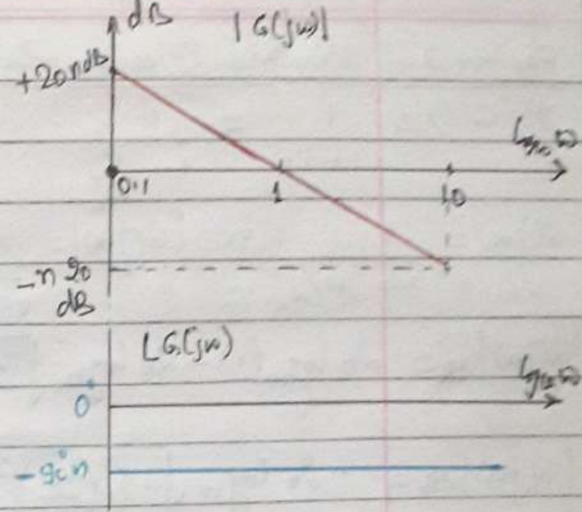
$G(j\omega) = 1/(0+j\omega)^n$

$|G(j\omega)| = 1/(\sqrt{0^2+\omega^2})^n = 1/\omega^n$

$|G(j\omega)|_{dB} = 20\log_{10}(1) - n\,20\log_{10}\omega$

$|G(j\omega)|_{dB} = -n\,20\log_{10}\omega$

$\angle G(j\omega) = -n\tan^{-1}(\omega/0) = -n\tan^{-1}(\infty) = -n\,90^\circ$




4) $G(s) = K/s$

$G(j\omega) = \dfrac{K+j0}{0+j\omega}$ ; $|G(j\omega)| = \dfrac{\sqrt{K^2+0^2}}{\sqrt{0^2+\omega^2}} = \dfrac{K}{\omega}$

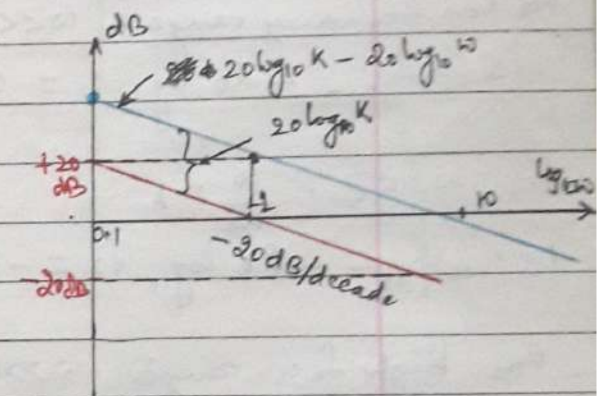
$|G(j\omega)|_{dB} = +20\log_{10}K - 20\log_{10}\omega$

$|G(j\omega)|_{dB} = -20\log_{10}\omega + 20\log_{10}K$

$\hookrightarrow y = m\,x + c$

$\therefore 20\log_{10}K = c$ or $\log_{10}K = c/20$ or $\boxed{K = (10)^{c/20}}$

$\angle G(j\omega) = -\tan^{-1}\left(\frac{\omega}{0}\right) = -\tan^{-1}(\infty) = -90^\circ$

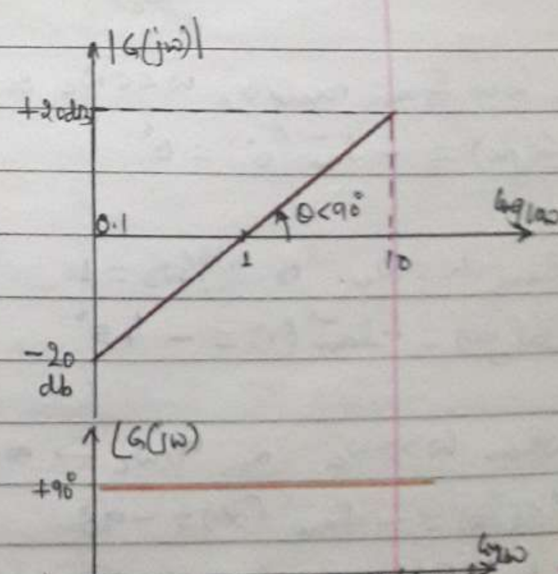



5) $G(s) = s$

$G(j\omega) = (0+j\omega)$ ; $|G(j\omega)| = \sqrt{0^2+\omega^2} = \omega$

$|G(j\omega)|_{dB} = 20\log_{10}\omega$, with $20 \to m$, $\log_{10}\omega \to x$

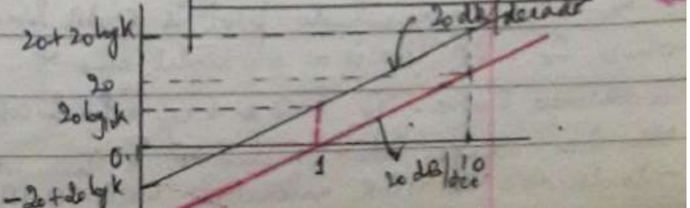
$\angle G(j\omega) = \tan^{-1}\left(\frac{\omega}{0}\right) = \tan^{-1}(\infty) = 90^\circ$




6) $G(s) = Ks$ ; $|G(j\omega)| = K\sqrt{0^2+\omega^2} = K\omega$

$|G(j\omega)|_{dB} = 20\log_{10}\omega + 20\log_{10}K$, with $20 \to m$, $\log_{10}\omega \to x$, $20\log_{10}K \to c$

$\angle G(j\omega) = \tan^{-1}\left(\frac{\omega}{0}\right) = \tan^{-1}(\infty) = 90^\circ$



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Factor 3:- Simple Poles or Zeros $(1+Ts)^{\pm 1}$

7) $G(s) = \frac{1}{1+Ts} = \frac{1}{1+\frac{s}{\omega_c}}$

$G(j\omega) = \frac{1}{1+j\omega/\omega_c}$ or $|G(j\omega)| = \frac{1}{\sqrt{1+(\omega/\omega_c)^2}}$

$|G(j\omega)|_{dB} = 20\log_{10}(1) - 20\log_{10}\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2}$

$|G(j\omega)|_{dB} = -20\log_{10}\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2}$

For Low frequency range - $\omega << \omega_c$ or $\omega/\omega_c << 1$

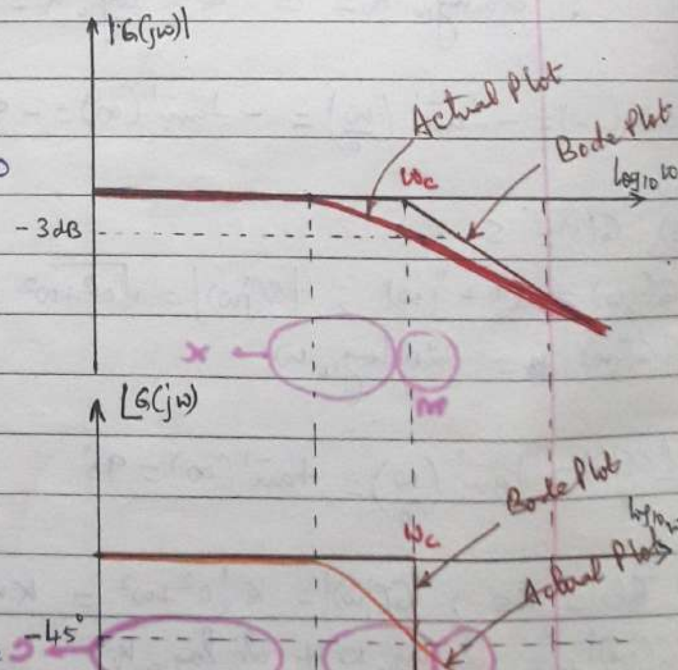
$\therefore |G(j\omega)|_{dB} = -20\log_{10} 1 = 0$ dB [for low freq. it is a st. line of 0 dB only]

for $\omega = \omega_c$, $\frac{\omega}{\omega_c} = 1$

$|G(j\omega)|_{dB} = -20\log_{10}\sqrt{2} = -3$ dB } for $\omega/\omega_c = 2$, $|G(j\omega)|_{dB} = -7$ dB

for High freq. range $\omega >> \omega_c$ or $\omega/\omega_c >> 1$

or $|G(j\omega)|_{dB} = -20\log_{10}\left(\frac{\omega}{\omega_c}\right) = -20\log_{10}\omega + 20\log_{10}\omega_c$




Now $\angle G(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_c}\right)$

In low freq. range, $\omega << \omega_c$ or $\frac{\omega}{\omega_c} \to 0$

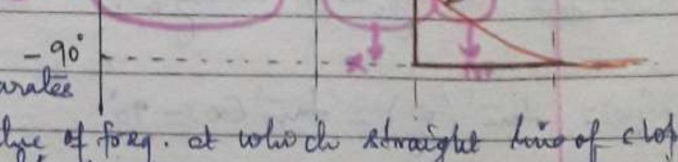
$\angle G(j\omega) = -\tan^{-1}(0^\circ) = 0^\circ$

When $\omega = \omega_c$ or $\omega/\omega_c = 1$

$\angle G(j\omega) = -\tan^{-1}(1) = -45^\circ$

When $\omega >> \omega_c$ or $\omega/\omega_c \to \infty$

$\angle G(j\omega) = -\tan^{-1}(\infty) = -90^\circ$




→ for LFR, magnitude plot is 0 dB line while for HFR it is a straight line of -20 dB/dec slope. To find a freq. which separates the freq. range into LF and HF is the value of freq. at which straight line of slope -20 dB/dec in HFR intersects with 0 dB line in LFR.

So, such intersection takes place when $-20\log_{10}\frac{\omega}{\omega_c} = 0\ dB$

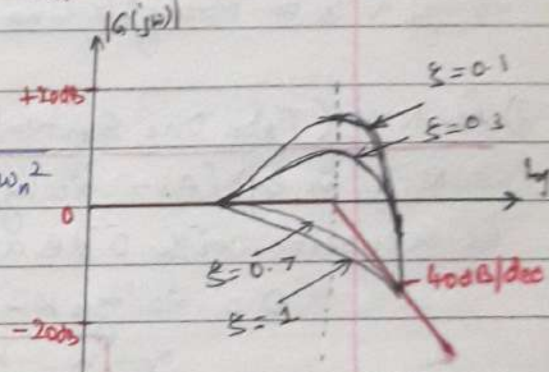
$$\log_{10}\frac{\omega}{\omega_c} = 0 \quad \text{or} \quad \frac{\omega}{\omega_c} = 1$$

$$\text{or} \quad \boxed{\omega = \omega_c = \text{Corner frequency}}$$

- The frequency at which the slope changes from 0 dB to −20dB/decade is called corner frequency, denoted by $\omega_c$.

Factor 4:- Quadratic Pole or Zero $\left(1 + \frac{2\xi s}{\omega_n} + \frac{s^2}{\omega_n^2}\right)$ -

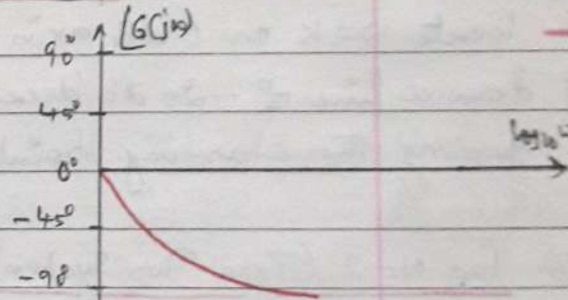
$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}. \quad \text{Now put } s = j\omega$$




$$G(j\omega) = \frac{\omega_n^2}{(j\omega)^2 + 2\xi\omega_n(j\omega) + \omega_n^2} = \frac{\omega_n^2}{-\omega^2 + j2\xi\omega_n\omega + \omega_n^2}$$

$$G(j\omega) = \frac{\omega_n^2}{\omega_n^2 - \omega^2 + j2\xi\omega_n\omega} = \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + j2\xi\frac{\omega}{\omega_n}}$$

→ $|G(j\omega)|_{dB} = -20\log_{10}\sqrt{\left\{1 - \left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + 4\xi^2\left(\frac{\omega}{\omega_n}\right)^2}$




Let $\omega/\omega_n = u$

$$\therefore |G(j\omega)|_{dB} = -20\log_{10}\sqrt{(1-u^2)^2 + 4\xi^2u^2}$$

Now when $u \ll 1$ i.e. $\frac{\omega}{\omega_n} \ll 1$

$$|G(j\omega)|_{dB} = -20\log_{10}\sqrt{1} = 0\ dB$$

when $u \gg 1$ i.e. $\omega/\omega_n \gg 1$ then $|G(j\omega)|_{dB} = -20\log_{10}u^2 = -40\log_{10}u$

→ $\angle G(j\omega) = -\tan^{-1}\frac{2\xi u}{1-u^2}$, then following conditions will arise-

when $u \ll 1$ i.e. $\omega/\omega_n \ll 1$ → $\angle G(j\omega) = -\tan^{-1}2\xi u = -\tan^{-1}(0) = 0°$

when $u \gg 1$, i.e. $\omega/\omega_n \gg 1$ → $\angle G(j\omega) = \tan^{-1}2\xi/u$

when $u = 1$, $\angle G(j\omega) = -\tan^{-1}(\infty) = -90°$

→ Hence general mag. plot for quadratic factor is 0dB line till corner frequency and then straight line of slope −40dB/dec

To find corner freq $-40\log\frac{\omega}{\omega_n} = 0\ dB$ or $\frac{\omega}{\omega_n} = 1$ or $\boxed{\omega_c = \omega_n}$

## Initial Slope of Bode Plot -

Let $G(s)H(s) = \frac{K}{s^N}$ now put $s = j\omega$

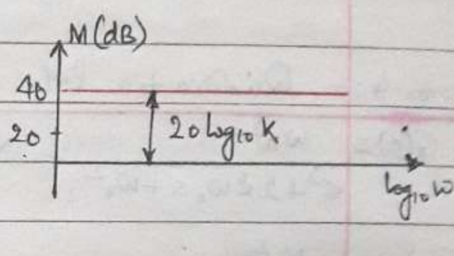
$\therefore G(j\omega)H(j\omega) = K/(j\omega)^N$

$$20\log_{10}|G(j\omega)H(j\omega)| = 20\log_{10}\left|\frac{K}{(j\omega)^N}\right| = 20\log_{10}K - 20N\log_{10}\omega \quad \text{—} (A)$$

1) for $N = 0$ (Type Zero System) -

$$20\log_{10}|G(j\omega)H(j\omega)| = 20\log_{10}K$$

$20\log_{10}K$ is a straight line



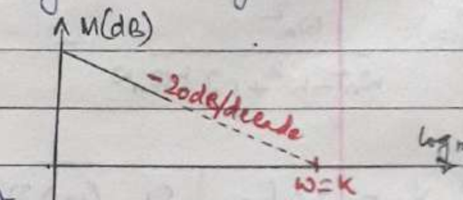

2) for $N = 1$ (Type One System) -

Put $N = 1$ in eq$^n$ (A) - $20\log_{10}|G(j\omega)H(j\omega)| = 20\log_{10}K - 20\log_{10}\omega$

intersection with 0 dB axis - slope

$$0 = 20\log_{10}K - 20\log_{10}\omega$$

or $\boxed{\omega = K}$




- locate $\omega = K$ on 0 dB axis and at this point draw a line of $-20$ dB/decade produce it till it intersects the y-axis giving the starting point on Bode Plot.

3). for $N = 2$ (Type Two System) -

Put $N = 2$ in eq$^n$ (A) - $20\log_{10}K - 40\log_{10}\omega = 20\log_{10}|G(j\omega)H(j\omega)|$

Intersection with 0 dB axis - slope

$$0 = 20\log_{10}K - 40\log_{10}\omega \quad \text{or} \quad 20\log_{10}K = 40\log_{10}\omega$$

$$20\log_{10}K = 2 \cdot 20\log_{10}\omega = 20\log_{10}\omega^2$$

or $\omega^2 = K$ or $\boxed{\omega = \sqrt{K}}$

- Hence graph intersects 0 dB axis at $\omega = \sqrt{K}$. Locate $\omega = \sqrt{K}$ on 0 dB axis and draw a line of $-40$ dB/decade & produce it to y-axis which is the starting point.

| Type of system | Initial slope 0dB axis | Intersection with. |
|---|---|---|
| 0 | 0dB/decade | Parallel to 0 dB axis |
| 1 | $-20$ dB/decade | $K$ |
| 2 | $-40$ dB/decade | $K^{1/2}$ |
| 3 | $-60$ dB/decade | $K^{1/3}$ |
| ⋮ | ⋮ | ⋮ |
| N | $-20N$ dB/decade | $K^{1/N}$ |

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## Procedure for Drawing Bode Plots –

Consider the transfer function –

$$\rightarrow G(s) = \frac{K(1+sT_a)(1+sT_b)\ldots}{s^N(1+sT_1)(1+sT_2)\ldots\left[1+2\xi\left(\frac{\omega}{\omega_n}\right)+\left(j\frac{\omega}{\omega_n}\right)^2\right]} \quad \text{—(1)}$$

where N defines the no. of poles at the origin (Type of system)

Now put $s = j\omega$ in above transfer function

$$\rightarrow G(j\omega) = \frac{K(1+j\omega T_a)(1+j\omega T_b)\ldots}{(j\omega)^N(1+j\omega T_1)(1+j\omega T_2)\ldots\left[1+2\xi(\omega/\omega_n)+(j\omega/\omega_n)^2\right]} \quad \text{—(2)}$$

$$\rightarrow 20\log_{10}|G(j\omega)| = 20\log_{10}K + 20\log_{10}\sqrt{1+\omega^2T_a^2} + 20\log_{10}\sqrt{1+\omega^2T_b^2} + \ldots - 20N\log_{10}\omega$$
$$- 20\log_{10}\sqrt{1+\omega^2T_1^2} - 20\log_{10}\sqrt{1+\omega^2T_2^2} - 20\log_{10}\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+4\xi^2\left(\frac{\omega}{\omega_n}\right)^2} \quad \text{—(3)}$$

$$\rightarrow \text{Phase Angle } \angle G(j\omega) = \tan^{-1}\omega T_a + \tan^{-1}\omega T_b - N(90^\circ) - \tan^{-1}\omega T_1 - \tan^{-1}\omega T_2 \ldots$$
$$- \tan^{-1}\left[\frac{2\xi\omega\omega_n}{\omega_n^2-\omega^2}\right] \quad \text{—(4)}$$

**Step 1:-** Identify the corner frequency

**Step 2:-** Draw the asymptotic magnitude plot. The slope will change at each corner frequency by +20dB/dec for zero and by −20dB/dec for pole. For complex conjugate pole and zero the slope will change by ± 40 db/dec.

**Step 3:** i) For type '0' system draw a line upto first (lowest) corner frequency having 0 db/decade slope.

ii) For type '1' system draw a line having slope −20 db/decade upto $\omega = K$. Mark lowest corner frequency first.

iii) For type '2' system draw a line having slope −40db/dec upto $\omega = \sqrt{K}$ and so on. Mark lowest corner freq. first.

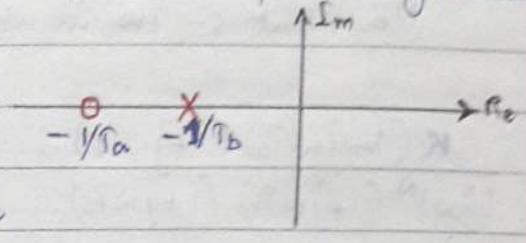
**Step 4:** Draw a line upto second corner frequency by adding the slope of the next pole or zero to the previous slope and so on.

**Step 5:-** Calculate phase angle for different values of ω from eqⁿ (4) and join all the points.

## Minimum and Non-Minimum Phase Systems-

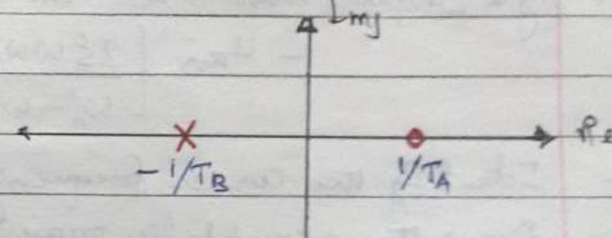
- The transfer func$^n$s having no poles and zeros in the RH of s-plane are called min$^m$ phase T'fer functions. Systems with minimum phase transfer func$^n$s are called minimum phase systems.

$$G_1(j\omega) = \frac{1+j\omega T_a}{1+j\omega T_b}$$ where pole is at $s = -1/T_b$ and zero at $-1/T_a$

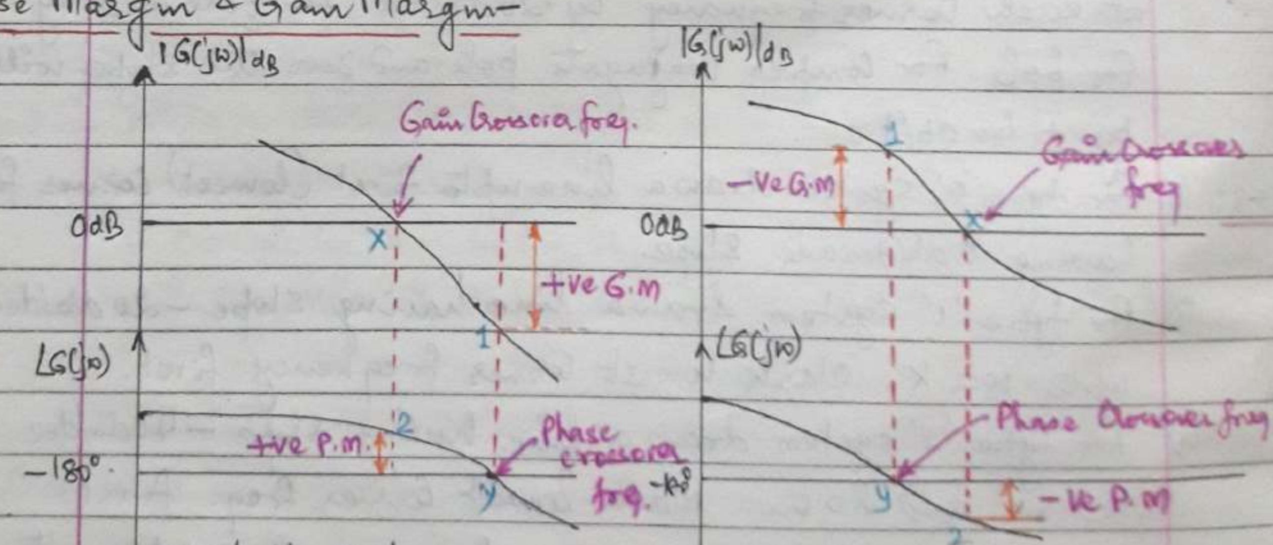



- The transfer func$^n$s having poles and/or zeros in the RH of s-plane are called non-minimum phase transfer functions. Systems with non-minimum phase transfer func$^n$s are called non-minimum phase systems.

$$G_2(j\omega) = \frac{1-j\omega T_A}{1+j\omega T_B}$$




## Phase Margin & Gain Margin-




a) Stable system b) Unstable system

- The point at which the magnitude curve crosses the 0dB line is the gain crossover frequency. The point where the phase curve crosses the 180° line is called phase crossover frequency.

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**Gain Margin** – Gain margin is defined as the margin in gain allowable by which gain can be increased till system reaches on the verge instability.

- Mathematically gain margin is defined as the reciprocal of the magnitude of the $G(j\omega)H(j\omega)$ at phase-crossover frequency.

$$\boxed{G.M. = \frac{1}{|G(j\omega)H(j\omega)|_{\omega=\omega_{c2}}}}$$ in Decibels

where $\omega_{c2}$ = Phase cross over frequency

In decibels $G.M. = 20\log_{10}\frac{1}{|G(j\omega)H(j\omega)|_{\omega=\omega_{c2}}}$

or $$\boxed{G.M. = -20\log_{10}|G(j\omega)H(j\omega)|_{\omega=\omega_{c2}}}$$

**Phase Margin** – The amount of additional phase lag which can be introduced in the system till it reaches on the verge of instability is called phase margin (P.M.)

$$P.M. = \left[\angle G(j\omega)H(j\omega)_{\omega=\omega_{c1}}\right] - (-180^\circ)$$

or $$\boxed{P.M. = 180^\circ + \angle G(j\omega)H(j\omega)\Big|_{\omega=\omega_{c1}}}$$

where $\omega_{c1}$ = Gain cross-over freq.

- Positive gain margin means the system is stable and negative G.M. means system is unstable.
- for minimum phase system both phase margin and gain margin must be positive for the system to be stable.

Q1. Sketch the Bode Plot for the T.F. $G(s) = \frac{1000}{(1+0.1s)(1+0.001s)}$

Determine a) P.M b) G.M. c) Stability of system

Solution – Step 1 – Put $s=j\omega$ in the transfer function –

$$G(j\omega) = \frac{1000}{(1+0.1j\omega)(1+j0.001\omega)}$$

- System is type '0' hence initial slope of the Bode Plot 0dB/decade.

Starting point is $20\log_{10}K = 20\log_{10}1000 = 60\,db$.

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Corner frequencies, $\omega_1 = \frac{1}{0.1} = 10$ rad/sec

and $\omega_2 = \frac{1}{0.001} = 1000$ rad/sec

Step 2:- Mark the starting point 60dB on y-axis and draw a line of slope 0dB/decade upto first corner frequency.

Step 3:- From first corner frequency to second corner frequency draw a line with slope ($0-20 = -20$ dB/decade)

Step 4:- From 2nd corner freq. to next corner freq. (if given) draw a line with slope $-20+(-20) = -40$ db/decade.

Step 5:- Magnitude plot is complete and now phase plot will be drawn by calculating the phase at various frequencies.

Step 6:- $\phi = -\tan^{-1}(0.1\omega) - \tan^{-1}(0.001\omega)$

| $\omega$ | $\phi$ |
|---|---|
| 1 | $-5.76°$ |
| 10 | $-45.57°$ |
| 50 | $-81.55°$ |
| 100 | $-90°$ |
| 500 | $-115.4°$ |
| 1000 | $-134.42°$ |
| 2000 | $-153.15°$ |
| 5000 | $-168.57°$ |
| 8000 | $-172.79°$ |

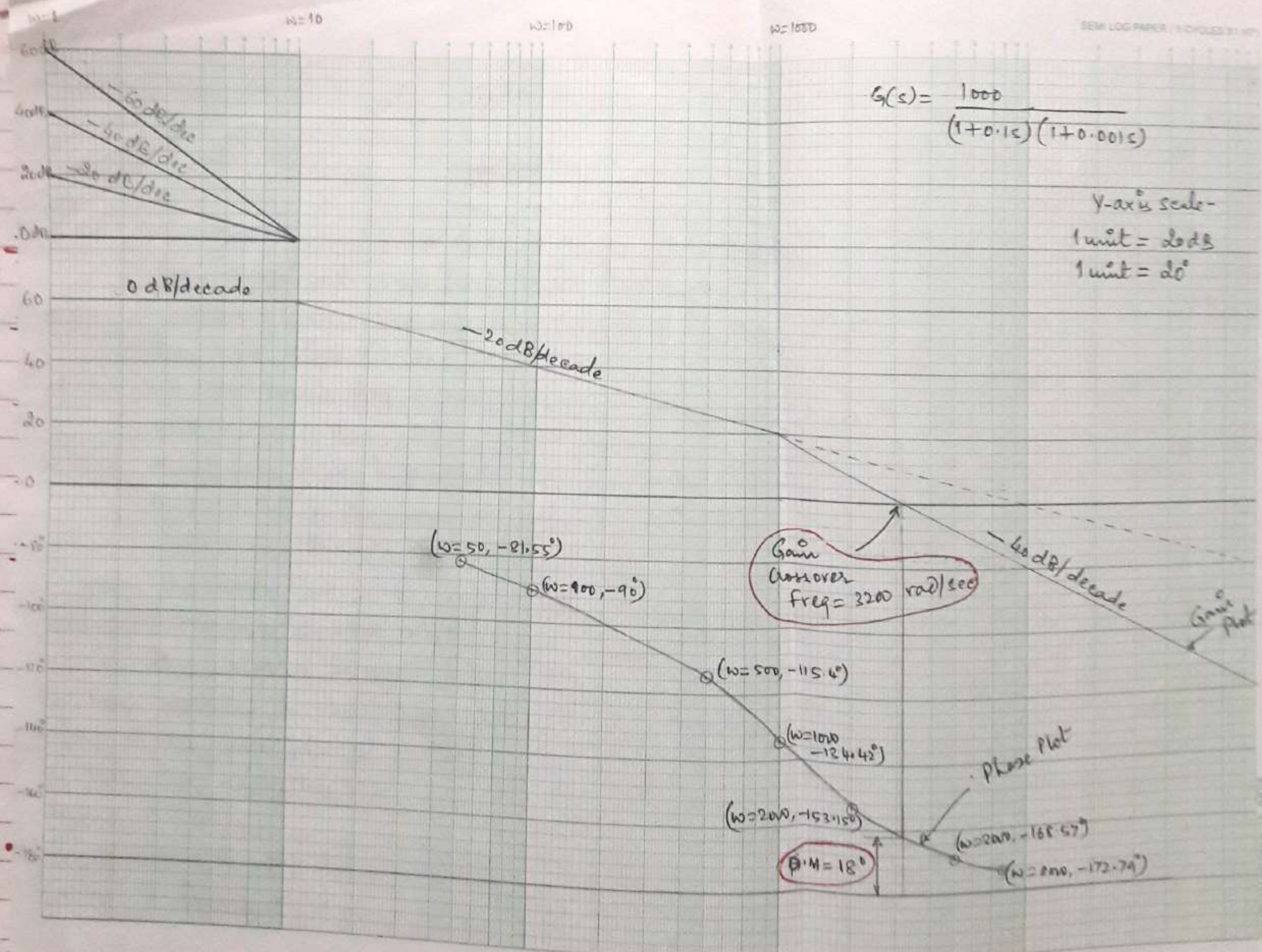
From the Bode Plot,

Gain cross-over freq = 3200 rad/sec and from this point a straight line has been dropped to phase plot, which intersects the phase plot at $-162°$

$\therefore$ P.M $= 180° - (162°) = 18°$

and G.M $= \infty$

- Since G.M. $= \infty$ and P.M. $= +18°$ the system is inherently stable



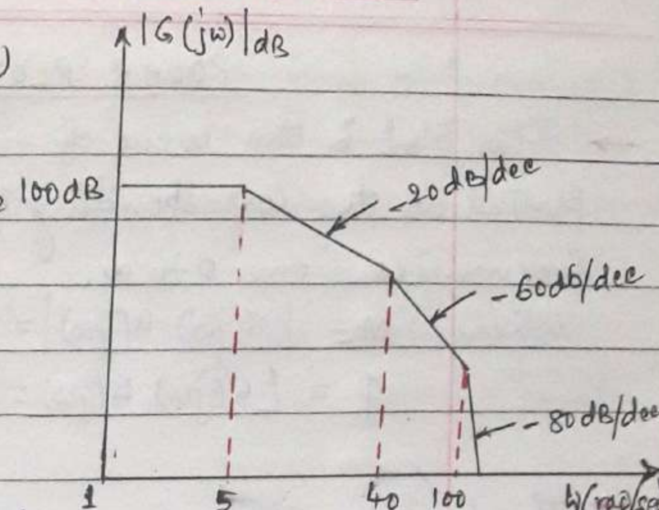
$G(s) = \frac{1000}{(1+0.1s)(1+0.001s)}$
Y-axis scale-
1 unit = 20 dB
1 unit = 20°
0 dB/decade
−20 dB/decade
−40 dB/decade
−60 dB/dec
−40 dB/dec
−20 dB/dec
ω=1
ω=10
ω=100
ω=1000
(ω=50, −81.55°)
(ω=400, −90°)
(ω=500, −115.6°)
(ω=1000, −124.42°)
(ω=2000, −153.15°)
(ω=2000, −168.57°)
(ω=8000, −172.74°)
Gain Crossover freq= 3200 rad/sec
P.M = 18°
Phase Plot
Gain Plot

6

Q4. The magnitude plot of the O.L.T.F. $G(s)$ of a certain system is shown –

a) Determine $G(s)$ if it is known that the system is of minm phase type.

b) Estimate the phase at each of the corner frequencies.




Soln – Since given system is of minm phase type means it has no poles or zeros in the RH side of s-plane.

– At $\omega = 5$ rad/sec slope changes to $-20$ db/dec which indicates a term $\left(1+\frac{s}{5}\right)$ or $(1+0.2s)$ in the denominator.

– At $\omega = 40$, slope changes to $-60$ dB/dec which is a net change of $-40$ dB/dec indicating a term $\left(1+\frac{s}{40}\right)^2$ in the denominator.

– At $\omega = 100$, slope changes to $-80$ db/dec, a net slope change of $-20$ db/dec. indicating a denominator term of $\left(1+\frac{s}{100}\right)$ or $(1+0.01s)$

– From 1 to 5 rad/sec, slope is 0 dB or $20\log_{10} K = 100$ or $K = 10^5$

$$\therefore \; G(s) = \frac{10^5}{(1+0.20s)(1+0.025s)^2(1+0.01s)}$$

Put $s = j\omega$ $\therefore$ $$G(j\omega) = \frac{10^5}{(1+j0.20\omega)(1+j0.025\omega)(1+j0.01\omega)}$$

$$\phi = -\tan^{-1}(0.2\omega) - 2\tan^{-1}(0.025\omega) - \tan^{-1}(0.01\omega)$$

At $\omega = 5$, $\phi = -\tan^{-1}(0.2\times 5) - 2\tan^{-1}(0.025\times 5) - \tan^{-1}(0.01\times 5) = -62.11^\circ$

At $\omega = 40$, $\phi = \tan^{-1}(0.2\times 40) - 2\tan^{-1}(0.025\times 40) - \tan^{-1}(0.01\times 40) = -194.67^\circ$

At $\omega = 100$ $\phi = \tan^{-1}(0.2\times 100) - 2\tan^{-1}(0.025\times 100) - \tan^{-1}(0.01\times 100) = -268.53^\circ$

# POLAR PLOT

→ Polar plot is the locus of tips of the phasors of various magnitudes plotted at the corresponding phase angles for different values of frequencies from 0 to ∞.

where $M = |G(j\omega)H(j\omega)|$ = Magnitude

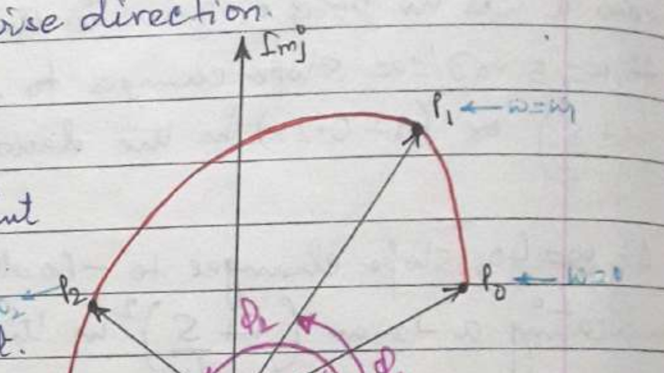
$\phi = \angle G(j\omega)H(j\omega)$ = Phase

→ The positive angles are measured in anticlockwise direction while negative are measured in clockwise direction.

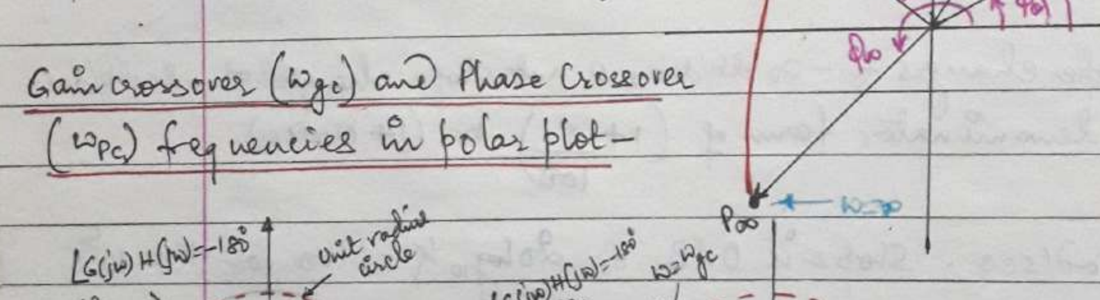
→ So $\omega \to 0$ $M_0 \angle \phi_0$ Starting Point

$\omega \to \infty$ $M_\infty \angle \phi_\infty$ Terminating Point

$\phi_\infty - \phi_0$ = Rotation of starting point to reach to the terminating point.




Gain crossover ($\omega_{gc}$) and Phase Crossover ($\omega_{pc}$) fequencies in polar plot–




a) $\omega_{pc} > \omega_{gc}$ (GM & PM +ve – stable) b) $\omega_{gc} > \omega_{pc}$ (GM & PM –ve → Unstable)

– In Bode Plot the frequency at which $|G(j\omega)H(j\omega)| = 0$ dB is called gain crossover frequency. But in polar plot dB values are not used.

– So $|G(j\omega)H(j\omega)| = 1$ corresponding to 0 dB at $\omega = \omega_{gc}$ from polar plot point of view. But location of a point with M = 1 is important.

- Consider a polar plot, to get a point with $M=1$, draw a circle with radius 1 and centre as origin. The point where this circle intersects polar plot is the point where $|G(j\omega)H(j\omega)| = 1$ and corresponding frequency is $\omega = \omega_{gc}$

- Now $\omega_{pc}$ is the frequency at which $\angle G(j\omega)H(j\omega) = -180°$. In Polar plot the point $\angle G(j\omega)H(j\omega) = -180°$ is a point on the -ve real axis. Such a point Q is shown.

- In stability determination, $|G(j\omega)H(j\omega)| = 1$ & $\angle G(j\omega)H(j\omega) = -180°$ is nothing but a point $-1+j0$ on the -ve real axis and is called Critical point in Polar and Nyquist plots.

- In Polar plots P.M. can't be obtained accurately but G.M. can be because it is the intersection of polar plot with negative real axis. Mathematically it can be obtained as follows; as $\omega = \omega_{pc}$ is such a frequency at which imaginary part of $G(j\omega)H(j\omega)$ becomes zero, when $G(j\omega)H(j\omega)$ is expressed in rectangular coordinates.

a) Rationalize the O.L.T.F. $G(j\omega)H(j\omega)$

b) Separate the real and imaginary parts of $G(j\omega)H(j\omega)$, both a func's of $\omega$.

c) Equate imaginary part to zero to get equation as $f(\omega) = 0$. Solve this to get value of $\omega$ which is making this imaginary part zero, i.e. $\omega = \omega_{pc}$ this frequency should be positive finite and greater than zero. Otherwise it can be concluded that there is no intersection of polar plot with the negative real axis.

d) Substitute this value of $\omega_{pc}$ in the real part to get actual co-ordinates of an intersection of point of polar plot with -ve real axis.

Steps to draw Polar Plot -

Step 1: Determine the T.F. $G(s)$

Step 2: Put $s = j\omega$ in the $G(s)$ and write system eq$^n$ in polar form $|G(j\omega)| \angle G(j\omega)$

Step 3: At $\omega = 0$ and $\omega = \infty$ find $|G(j\omega)|$ by $\lim_{\omega \to 0} |G(j\omega)|$ & $\lim_{\omega \to \infty} |G(j\omega)|$

Step 4:- At $\omega=0$ and $\omega=\infty$, find $\angle G(j\omega)$ by $\lim_{\omega \to 0} \angle G(j\omega)$ & $\lim_{\omega \to \infty} \angle G(j\omega)$

Step 5:- Rationalize the func$^n$ $G(j\omega)$ and separate the real and imaginary parts.

Step 6:- Put $Re[G(j\omega)]=0$, determine the frequency at which plot intersects the Imaginary axis and calculate the intersection value by putting the above calculated frequency in $G(j\omega)$

Step 7:- Put $Im[G(j\omega)]=0$, determine the frequency at which plot intersects the real axis and calculate intersection value by putting the above calculated frequency in $G(j\omega)$

Step 8:- Sketch the Polar plot with the help of above information.

Q1. Draw the polar plot for the given transfer function-

$$G(s)\,H(s) = \frac{1}{s(1+Ts)}$$

Sol$^n$ → frequency domain transfer function is given by-

$$G(j\omega)\,H(j\omega) = \frac{1}{(j\omega)\,(1+j\omega T)} = \frac{(1+j0)}{(0+j\omega)\,(1+j\omega T)}$$

Now → $$|G(j\omega)\,H(j\omega)| = \frac{1}{\omega\sqrt{1+\omega^2T^2}} \quad \text{--- (1)}$$

$$\angle G(j\omega)\,H(j\omega) = \frac{\tan^{-1}(0/1)}{\tan^{-1}(\omega/0)\cdot\tan^{-1}(\omega T)} = \frac{0^\circ}{90^\circ,\ \tan^{-1}\omega T}$$

$$\rightarrow \angle G(j\omega)\,H(j\omega) = -90^\circ - \tan^{-1}(\omega T) \quad \text{--- (2)}$$

→ Now at $\omega=0$, $\lim_{\omega \to 0} |G(j\omega)H(j\omega)| = \infty$

and $\lim_{\omega \to 0} \angle G(j\omega)\,H(j\omega) = -90^\circ - (\tan^{-1} 0) = -90^\circ$

→ At $\omega=\infty$, $\lim_{\omega \to \infty} |G(j\omega)\,H(j\omega)| = 0$

and $\lim_{\omega \to \infty} \angle G(j\omega)\,H(j\omega) = -90^\circ - \tan^{-1}(\infty) = -180^\circ$

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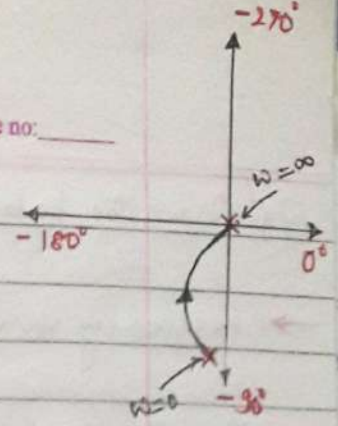
→ Rotation of plot $= \phi_\infty - \phi_0 = -180^\circ - (-90^\circ) = -90^\circ$




→ Now rationalising the transfer function -

$$\frac{1}{j\omega(1+j\omega T)} \times \frac{-j\omega(1-j\omega T)}{-j\omega(1-j\omega T)} = \frac{-j\omega - \omega^2 T}{\omega^2(1+\omega^2T^2)}$$

$$\Rightarrow \frac{-\cancel{\omega^2}T}{\cancel{\omega^2}(1+\omega^2T^2)} - \frac{j\omega}{\omega^2(1+\omega^2T^2)} = 0$$

or $\frac{-\omega^2 T}{\omega^2(1+\omega^2T^2)} = 0$ or $\omega = \infty$ (No non zero real value) } for $\omega=\infty$, the real & imag. axis are intersected at $0\angle -180^\circ$

and $\frac{-j\cancel{\omega}}{\omega^{\cancel{2}}(1+\omega^2T^2)} = 0$ or $\omega=\infty$ (No non zero real value)

- which means the plot is neither intersecting real nor imaginary.

Q2. Draw the polar plot for the given T.F. - $G(s)H(s) = \frac{1}{s^2(1+Ts)}$.

→ Frequency domain transfer func$^n$ is $G(j\omega)H(j\omega) = \frac{(1+j0)}{(j\omega)^2(1+j\omega T)}$

→ Magnitude $|G(j\omega)H(j\omega)| = \frac{1+j0}{\omega^2\sqrt{1+\omega^2T^2}}$

→ Phase angle $\angle G(j\omega)H(j\omega) = \tan^{-1}(0/1) - \tan^{-1}(\omega/0) - \tan^{-1}\omega/0 - \tan^{-1}\omega T$

$\angle G(j\omega)H(j\omega) = -180^\circ - \tan^{-1}(\omega T)$

→ At $\omega = 0$

$\lim_{\omega\to 0} |G(j\omega)H(j\omega)| = \infty$

$\lim_{\omega\to 0} \angle G(j\omega)H(j\omega) = -180^\circ$

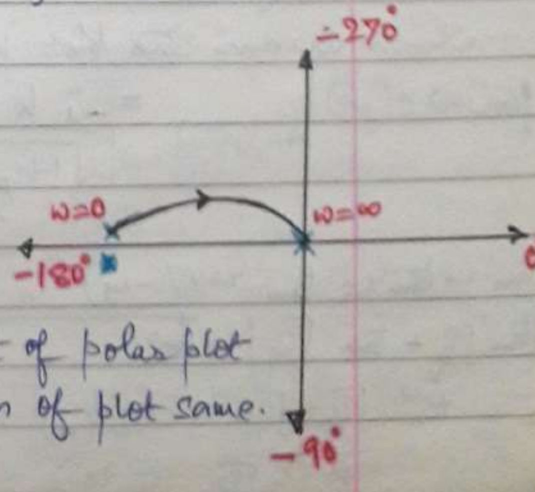
→ At $\omega=\infty$

$\lim_{\omega\to\infty} |G(j\omega)H(j\omega)| = 0$

$\lim_{\omega\to 0} \angle G(j\omega)H(j\omega) = -270^\circ$

→ Rotation of plot $= -270^\circ - (-180^\circ) = -90^\circ$

→ No intersection with either the real or imaginary axes.




* A pole at origin shifts the starting point of polar plot by 90° in CW direction keeping the rotation of plot same.

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Q3. Draw the polar plot for the given T.F. $G(s)H(s) = \dfrac{K}{(1+T_1 s)(1+T_2 s)}$

→ frequency domain T.F. is $G(j\omega)H(j\omega) = \dfrac{(K+j0)}{(1+j\omega T_1)(1+j\omega T_2)}$

→ Magnitude $|G(j\omega)H(j\omega)| = \dfrac{\sqrt{K^2+0^2}}{\sqrt{1^2+\omega^2T_1^2}\sqrt{1+\omega^2T_2^2}} = \dfrac{K}{\sqrt{1+\omega^2T_1^2}\sqrt{1+\omega^2T_2^2}}$

→ Angle $\angle G(j\omega)H(j\omega) = \dfrac{\tan^{-1}(0/K)}{\tan^{-1}(\omega T_1)\cdot\tan^{-1}(\omega T_2)}$

$$\phi = -\tan^{-1}(\omega T_1) - \tan^{-1}(\omega T_2)$$

→ At $\omega = 0$

$\lim_{\omega\to 0} |G(j\omega)H(j\omega)| = K$

$\lim_{\omega\to 0} \angle G(j\omega)H(j\omega) = 0^\circ$

→ At $\omega = \infty$

$\lim_{\omega\to\infty} |G(j\omega)H(j\omega)| = 0$

$\lim_{\omega\to\infty} \angle G(j\omega)H(j\omega) = -180^\circ$

→ Rationalizing the T.F. –

$$\frac{K}{(1+j\omega T_1)(1+j\omega T_2)} \times \frac{(1-j\omega T_1)(1-j\omega T_2)}{(1-j\omega T_1)(1-j\omega T_2)} = \frac{K\left[1-j\omega T_2 - j\omega T_1 - \omega^2 T_1 T_2\right]}{(1+\omega^2T_1^2)(1+\omega^2T_2^2)}$$

Separating the real and imaginary parts –

$$\frac{K(1-\omega^2T_1T_2) - jK\omega(T_1+T_2)}{(1+\omega^2T_1^2)(1+\omega^2T_2^2)} = \frac{K(1-\omega^2T_1T_2)}{(1+\omega^2T_1^2)(1+\omega^2T_2^2)} - \frac{jK\omega(T_1+T_2)}{(1+\omega^2T_1^2)(1+\omega^2T_2^2)} \quad (4)$$

→ Equating real part to zero –

$K(1-\omega^2T_1T_2) = 0$

$1-\omega^2T_1T_2 = 0$

or $\omega = 1/\sqrt{T_1T_2}$ → +ve Real nonzero value from equating real part to zero which means the plot will intersect the imaginary axis.

→ Equating imaginary part to zero

$-jK\omega(T_1+T_2) = 0$ ; $\omega = 0$

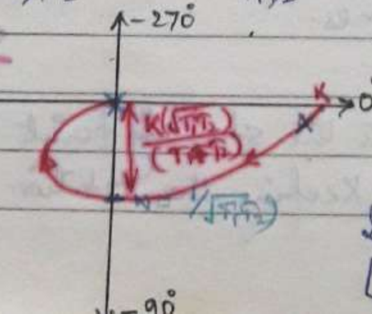
→ Rotation $= -180^\circ - (0^\circ) = -180^\circ$

$$|G(j\omega)H(j\omega)|_{\omega=1/\sqrt{T_1T_2}} = \left|\frac{-j\cdot K}{\sqrt{1+\frac{1}{T_1T_2}\times T_1^2}}\cdot\frac{1}{\sqrt{1+\frac{1}{T_1T_2}\cdot T_2^2}}\right| = \frac{K\sqrt{T_1T_2}}{(T_1+T_2)}\angle -90^\circ$$

$\tan^{-1}x + \tan^{-1}y = \tan^{-1}(x+y/1-xy)$

$= -\left[\tan^{-1}\sqrt{\frac{T_1}{T_2}} + \tan^{-1}\sqrt{\frac{T_2}{T_1}}\right]$

$= -\tan^{-1}\dfrac{\sqrt{\frac{T_1}{T_2}}+\sqrt{\frac{T_2}{T_1}}}{1-\sqrt{\frac{T_1}{T_2}}\sqrt{\frac{T_2}{T_1}}} = -\tan^{-1}(\infty) = -90^\circ$




$\angle G(j\omega)H(j\omega) = -\tan^{-1}\sqrt{\frac{T_1}{T_2}} - \tan^{-1}\sqrt{\frac{T_2}{T_1}}$

If $T_1 = T_2$ then

$\angle G(j\omega)H(j\omega) = -90^\circ$

# NYQUIST PLOT

Steps for drawing Nyquist Plot-

- Draw the polar plot by the conventional technique.
- Then draw the mirror image of the polar plot.
- Closing of the plot from $\omega = 0^-$ to $0^+$
- closing should be clockwise in direction
- Radius should be infinite
- Closing takes place through $n\pi$ degrees, where $n$ = type of system
- No. of encirclements (ACW) of the point $(-1+j0)$ by $G(s)H(s)$ plot is given by

$$N = P_+ - Z_+$$

where $P+$ = No. of poles of $G(s)H(s)$ with +ve real part

$Z+$ = No. of Zeros of $1+G(s)H(s)=0$ with +ve real part.

- For a stable control system $Z_+ = 0$, there fore, the condition for a control system to be stable is

$$N = P_+ - 0 \quad \text{or} \quad N = P_+$$

- ACW encirclement is taken +ve while CW is taken -ve

Q1. Determine the closed loop stability of a control system whose O.L.T.F is

$$G(s)H(s) = \frac{K}{s(1+sT)}$$

Sol<sup></sup>- — frequency domain transfer func<sup></sup> is given by-

$$G(j\omega)H(j\omega) = \frac{K}{j\omega(1+j\omega T)}$$

- Magnitude is given by $|G(j\omega)H(j\omega)| = \dfrac{K}{\omega\sqrt{1+\omega^2T^2}} = M$

- Angle, $\phi = -90^\circ - \tan^{-1}(\omega T)$

- At $\omega = 0$

$$\lim_{\omega\to 0} |G(j\omega)H(j\omega)| = \infty$$

$$\lim_{\omega\to 0} \angle G(j\omega)H(j\omega) = -90$$

- At $\omega = \infty$

$$\lim_{\omega\to\infty} |G(j\omega)H(j\omega)| = 0$$

$$\lim_{\omega\to\infty} \angle G(j\omega)H(j\omega) = -180^\circ$$

- Rationalizing the denominator & separate the real & imag axis.

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$$\frac{K}{j\omega(1+j\omega T)} * \frac{-j\omega(1-j\omega T)}{-j\omega(1-j\omega T)} = \frac{K[-j\omega-\omega^2 T]}{\omega^2(1+\omega^2T^2)}$$

or $$-\frac{\omega^2 TK}{\omega^2(1+\omega^2T^2)} - j\frac{K\omega}{\omega^2(1+\omega^2T^2)} = -\frac{TK}{1+\omega^2T^2} - j\frac{K}{\omega(1+\omega^2T^2)}$$ [III Quadrant]

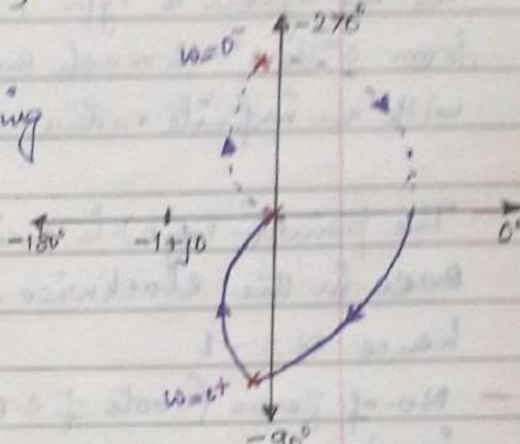
Equating real part to zero i.e. $-\dfrac{TK}{1+\omega^2T^2} = 0$ i.e. when $\omega = \infty$

Equating imaginary part to zero $-\dfrac{jK}{\omega^2(1+\omega^2T^2)} = 0$ when $\omega = \infty$

Means the plot terminates at $\omega = \infty \angle -90^\circ$

- Given system is a type '1' system hence closing takes place by $+\pi = 180^\circ$
- Now poles with +ve real part $N_+ = 0$ and the point $(-1+j0)$ is not encircled i.e. $N = 0$

$\therefore Z_+ = P_+ - N = 0$




- Since no. of zeros or roots of the C.E. with +ve real part is nil, so system is stable.

Q2. A closed loop control system is shown which is given by the T.F —
$G(s)H(s) = \dfrac{K}{s(sT-1)}$, determine the stability using Nyquist criteria

Sol$^n$ — Put $s = j\omega$ in the given T.F

$$G(j\omega)H(j\omega) = \frac{K}{j\omega(j\omega T-1)} \quad —(1)$$

- Magnitude, $M = \dfrac{K}{\omega\sqrt{1+\omega^2T^2}} = |G(j\omega)H(j\omega)|$

- Phase angle $\angle G(j\omega)H(j\omega) = -90^\circ - \tan^{-1}(-\omega T) = -90^\circ - [\pi - \tan^{-1}\omega T]$
$= \boxed{-90^\circ + 180^\circ + \tan^{-1}(\omega T)} = -270^\circ + \tan^{-1}(\omega T)$

$\dfrac{K}{j\omega(-1+j\omega T)} = \dfrac{0}{90^\circ - 180^\circ + \tan^{-1}(\infty)}$
$\to \tan^{-1}(\infty) = -90^\circ$

- At $\omega = 0$

$\lim_{\omega\to 0} |G(j\omega)H(j\omega)| = \infty$

$\lim_{\omega\to 0} \angle G(j\omega)H(j\omega) = -90^\circ + 180^\circ$
$= -270^\circ + 90^\circ$

- At $\omega = \infty$

$\lim_{\omega\to\infty} |G(j\omega)H(j\omega)| = 0$

$\lim_{\omega\to\infty} \angle G(j\omega)H(j\omega) = -270^\circ + 90^\circ = -180^\circ$

$-90^\circ + 180^\circ - (-90^\circ) = 180^\circ$

- Rotation of polar plot $= -180^\circ - (-270^\circ) = 90^\circ$

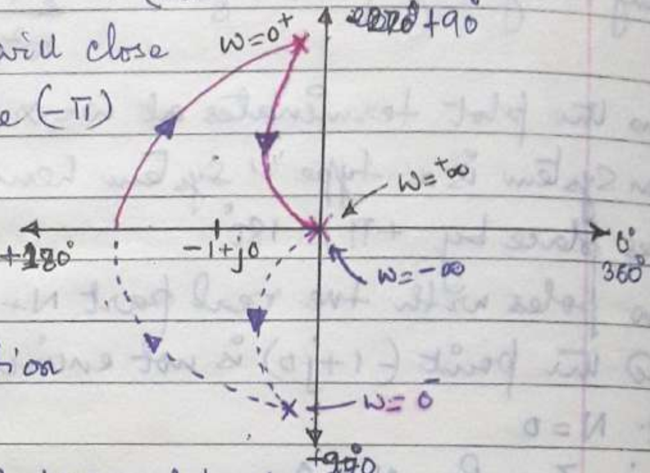
– Rationalizing eqⁿ (1) –

$$G(j\omega)H(j\omega) = \frac{K}{j\omega(j\omega T-1)} \times \frac{-j\omega(-1-j\omega T)}{-j\omega(-1-j\omega T)}$$

$$G(j\omega)H(j\omega) = \frac{K\left[j\omega - \omega^2 T\right]}{\omega^2\left[1+\omega^2T^2\right]} = \frac{jK\cancel{\omega}}{\omega^{\cancel{2}}(1+\omega^2T^2)} - \frac{\cancel{\omega^2}TK}{\cancel{\omega^2}\left[1+\omega^2T^2\right]}$$

$$G(j\omega)H(j\omega) = \frac{-TK}{1+\omega^2T^2} + \frac{jK}{\omega(\omega^2T^2+1)}$$ [$II^{nd}$ Quadrant]

– As the system is Type '1' plot will close from $0^-$ to $0^+$ through an angle $(-\pi)$ with an infinite radius




– The point $(-1+j0)$ is encircled once in the clockwise direction hence $N = -1$

– No. of zeros (roots of C.E.) with +ve real part is given by $N = P_+ - Z_+$ or $Z_+ = P_+ - N = 1-(-1) = 2$
[$K/s(sT-1)$ or $s = 1/T$ +ve real part of pole]

– Since there are two roots with +ve real parts hence system is unstable

Q3. Investigate the stability of control system whose O.L.T.F. is given by

$$G(s)H(s) = \frac{K}{s(sT_1+1)(sT_2+1)}$$

Solⁿ – Put $s=j\omega$ in $G(s)H(s)$

$$G(j\omega)H(j\omega) = \frac{K}{j\omega(1+j\omega T_1)(1+j\omega T_2)} \quad \text{—— (1)}$$

– $$|G(j\omega)H(j\omega)| = \frac{K}{\omega\sqrt{1+\omega^2T_1^2}\sqrt{1+\omega^2T_2^2}}$$

– $$\angle G(j\omega)H(j\omega) = -90^\circ - \tan^{-1}\omega T_1 + \tan^{-1}\omega T_2$$

– when $\omega \to 0$ ; $\lim_{\omega\to 0} |G(j\omega)H(j\omega)| = \infty$

$$\lim_{\omega\to 0} \angle G(j\omega)H(j\omega) = -90^\circ$$

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- when $\omega = \infty$ ; $\lim_{\omega \to \infty} |G(j\omega)H(j\omega)| = 0$ and $\lim_{\omega \to \infty} \angle G(j\omega)H(j\omega) = -270^\circ$

Rotation of plot (Polar) $= -270^\circ - (-90^\circ) = -180^\circ$

- Rationalizing the transfer function in freq. domain -

$$G(j\omega)H(j\omega) = \frac{K}{j\omega(1+j\omega T_1)(1+j\omega T_2)} \times \frac{-j\omega(1-j\omega T_1)(1-j\omega T_2)}{-j\omega(1-j\omega T_1)(1-j\omega T_2)}$$

$$G(j\omega)H(j\omega) = \frac{K(-j\omega - \omega^2 T_2 - \omega^2 T_1 + j\omega^3 T_1 T_2)}{\omega^2(1+\omega^2 T_1^2)(1+\omega^2 T_2^2)} = \frac{-K\omega^2(T_1+T_2) - j\omega K(1-\omega^2 T_1 T_2)}{\omega^2(1+\omega^2T_1^2)(1+\omega^2T_2^2)}$$

$$= \frac{-K\cancel{\omega^2}(T_1+T_2)}{\cancel{\omega^2}(1+\omega^2T_1^2)(1+\omega^2T_2^2)} - \frac{j\cancel{\omega}K(1-\omega^2T_1T_2)}{\omega^{\cancel{2}}(1+\omega^2T_1^2)(1+\omega^2T_2^2)}$$

$$= \frac{-K(T_1+T_2)}{(1+\omega^2T_1^2)(1+\omega^2T_2^2)} - \frac{jK(1-\omega^2T_1T_2)}{\omega(1+\omega^2T_1^2)(1+\omega^2T_2^2)} \quad \text{--- (2)}$$

(III Quadrant)

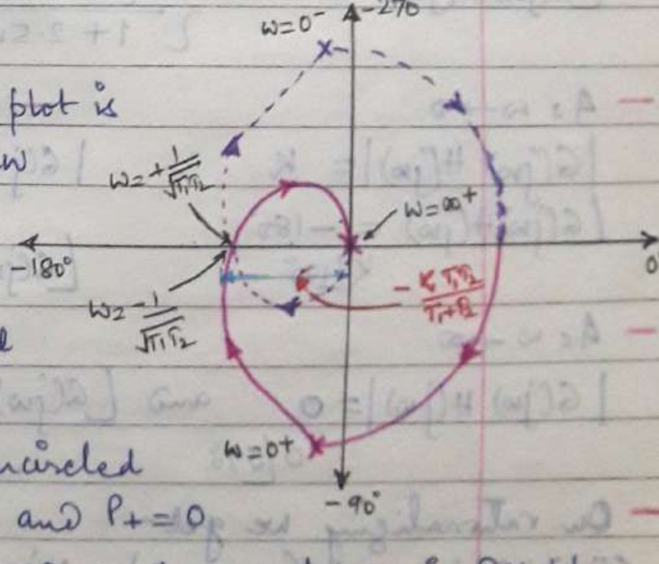
- Now equating imaginary part to zero, which will give intersection with real axis

$K(1-\omega^2T_1T_2) = 0$ or $1 = \omega^2 T_1 T_2$ or $\omega^2 = 1/T_1T_2$

or $\omega = \pm 1/\sqrt{T_1T_2}$ substitute this value in real part of (2)

$$|G(j\omega)H(j\omega)|_{\omega = 1/\sqrt{T_1T_2}} = \frac{-K(T_1+T_2)}{\left(1+\frac{1}{\cancel{T_1}T_2}\cdot T_1^{\cancel{2}}\right)\left(1+\frac{1}{T_1\cancel{T_2}}\cdot T_2^{\cancel{2}}\right)} = \frac{-K\cancel{(T_1+T_2)}\,T_1T_2}{\cancel{(T_1+T_2)}(T_1+T_2)}$$

$$|G(j\omega)H(j\omega)| = -K T_1 T_2 / T_1 + T_2$$




- As the system is type '1' hence the plot is closed from $\omega = 0^-$ to $0^+$ by $\pi$ in CW

- If $\frac{KT_1T_2}{T_1+T_2} < 1$ then the $-1+j0$ point is not encircled and system is stable.

- If $\frac{KT_1T_2}{T_1+T_2} > 1$ then $(-1+j0)$ is encircled twice thus $N = -2$ and $P_+ = 0$

$\therefore$ $Z_+ = P_+ - N$ or $Z_+ = 2$ or two roots are in RH plane thus making the system unstable.

$N = P_+ - Z_+$

$-2 = 0 - Z_+$ or $Z_+ = 2$

B and C and its encirclements is $-1$ (CW). Since $P_+ = 1$ therefore
$N = P_+ - Z_+$ or $-1 = 1 - Z_+$ or $Z_+ = 2$ (2 roots with +ve real Part hence unstable)

Q5. Determine the closed loop stability by using Nyquist Criteria for the OLTF

$$G(s) = \frac{(s+0.25)}{s^2(s+1)(s+0.5)}$$

Solⁿ – Put $s = j\omega$ in the given OLTF

$$G(j\omega) = \frac{(0.25+j\omega)}{(j\omega)^2(1+j\omega)(0.5+j\omega)}$$

– Magnitude of $G(j\omega)$

$$|G(j\omega)| = \frac{\sqrt{\omega^2+0.25^2}}{\omega^2\sqrt{1^2+\omega^2}\sqrt{\omega^2+0.5^2}} \quad \text{—(1)}$$

– Angle of $G(j\omega)$

$$\angle G(j\omega) = \tan^{-1}\left(\frac{\omega}{0.25}\right) - 180^\circ - \tan^{-1}\omega - \tan^{-1}\left(\frac{\omega}{0.5}\right)$$

$$\angle G(j\omega) = -180^\circ + \tan^{-1}(4\omega) - \left[\tan^{-1}\omega + \tan^{-1}2\omega\right]$$

$$= -180^\circ + \tan^{-1}(4\omega) - \tan^{-1}\frac{3\omega}{1-2\omega^2} = -180^\circ + \tan^{-1}\frac{4\omega - \frac{3\omega}{1-\omega^2}}{1+\frac{4\omega\cdot 3\omega}{1-\omega^2}}$$

$$= -180^\circ + \tan^{-1}\frac{\omega(1-8\omega^2)}{(1+10\omega^2)}$$

– Rationalizing $G(j\omega)$ –

$$G(j\omega) = \frac{(0.25+j\omega)}{(j\omega)^2(1+j\omega)(0.5+j\omega)} \times \frac{(-j\omega)(-j\omega)(1-j\omega)(0.5-j\omega)}{(-j\omega)(-j\omega)(1-j\omega)(0.5-j\omega)}$$

$$= \frac{-0.125\omega^2(1+10\omega^2) - j\,0.125\omega^3(1-8\omega^2)}{\omega^4(1+\omega^2)(0.25+\omega^2)}$$

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$$G(j\omega) = \frac{-0.125\omega^{2}(1+10\omega^2)}{\omega^{4}(1+\omega^2)(0.25+\omega^2)} - j\frac{0.125\omega^{3}(1-8\omega^2)}{\omega^4(1+\omega^2)(0.25+\omega^2)}$$

$$= \frac{-0.125(1+10\omega^2)}{\omega^2(1+\omega^2)(0.25+\omega^2)} - j\frac{0.125(1-8\omega^2)}{\omega(1+\omega^2)(0.25+\omega^2)}$$ (III Quad)

Equating imaginary part to zero [intersection with -ve real axis]

$$\frac{0.125(1-8\omega^2)}{\omega(1+\omega^2)(0.25+\omega^2)} = 0 \quad \text{or} \quad 1-8\omega^2=0$$

$$\text{or} \quad 1 = 8\omega^2$$

$$\text{or} \quad \omega^2 = 0.125 \quad \text{or} \quad \boxed{\omega = \pm\sqrt{0.125}}$$ — Intersection with the real axis

— when $\omega \to 0$

$$\lim_{\omega\to 0}|G(j\omega)| = \infty$$

$$\lim_{\omega\to 0}\angle G(j\omega) = -180^\circ$$

— At $\omega = \sqrt{1/8} = \sqrt{0.125}$

$$\lim_{\omega\to\sqrt{0.125}}|G(j\omega)| = 5.3$$

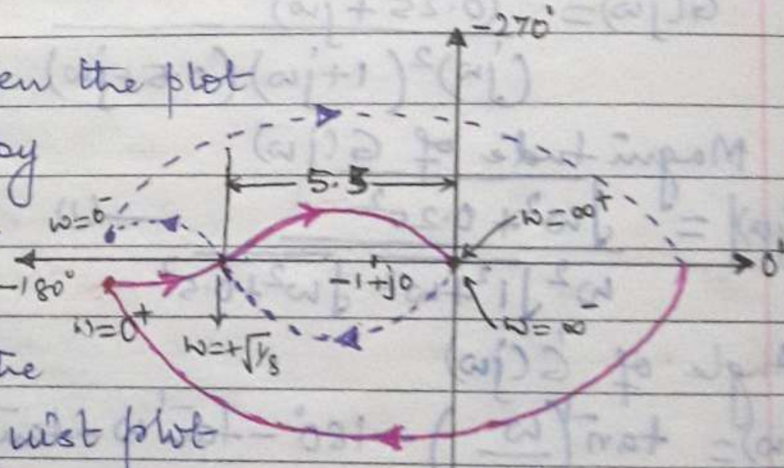
$$\lim_{\omega\to\sqrt{0.125}}\angle G(j\omega) = -180^\circ$$

— when $\omega \to \infty$

$$\lim_{\omega\to\infty}|G(j\omega)| = 0$$

$$\lim_{\omega\to\infty}\angle G(j\omega) = -270^\circ$$

— open loop T.F. is type '2', then the plot is closed from $\omega = 0^-$ to $0^+$ by an angle $-2\pi$ with $\infty$ radius




— No. of encirclements N of the point $(-1+j0)$ by the Nyquist plot is $-2$ (CW) and $P_+ = 0$

$$\therefore \quad Z_+ = 2$$

— closed loop C.E. has two roots with +ve real part thus the system is unstable.

Q.7. for a feedback control system, $G(s)H(s) = 40/(s+4)(s^2+2s+2)$. find gain margin and stability from Nyquist plot.

**Soln** – Put $s = j\omega$ in $G(s)H(s) = \dfrac{40}{(s+4)(s^2+2s+2)}$

$$G(j\omega)H(j\omega) = \frac{40}{(4+j\omega)(j^2\omega^2+j2\omega+2)} = \frac{40}{(4+j\omega)(-\omega^2+2+j2\omega)}$$

– $|G(j\omega)H(j\omega)| = \dfrac{40}{\sqrt{4^2+\omega^2}\sqrt{(2-\omega^2)^2+(2\omega)^2}}$

– $\angle G(j\omega)H(j\omega) = -\tan^{-1}\left(\dfrac{\omega}{4}\right) - \tan^{-1}\dfrac{2\omega}{2-\omega^2}$

* The quadratic factor contributes angle of $180^\circ$ as $\omega \to \infty$ and $0^\circ$ as $\omega \to 0$.

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- Rationalising $G(j\omega)H(j\omega)$

$$G(j\omega)H(j\omega) = \frac{40}{(4+j\omega)\{(2-\omega^2)+j2\omega\}} \times \frac{(4-j\omega)\{(2-\omega^2)-j2\omega\}}{(4-j\omega)\{(2-\omega^2)-j2\omega\}}$$

$$= \frac{40\{8-4\omega^2-j8\omega-j2\omega+j\omega^3-2\omega^2\}}{(4^2+\omega^2)\{(2-\omega^2)^2+(2\omega)^2\}}$$

$$= \frac{40\{8-6\omega^2\}}{D} + \frac{40j\omega\{\omega^2-10\}}{D} \quad \{1^{st}\ Quad\}$$

- Equating imaginary part to zero i.e. $\omega^2-10=0$ or $\omega^2=10$ or $\omega=\omega_{pc}=\sqrt{10}$ rad/sec.
- Equating real part $=0$, $\omega^2 = 8/6$ or $\omega = 2/\sqrt{3}$ when $\omega = 2/\sqrt{3}$, int. with imaginary $= j4.0$

- when $\omega \to 0$

$$\lim_{\omega\to 0}|G(j\omega)H(j\omega)| = \frac{40}{4\times 2} = 5$$

$$\lim_{\omega\to 0}\angle G(j\omega)H(j\omega) = 0^\circ$$

- when $\omega = \sqrt{10}$

$$\lim_{\omega\to\sqrt{10}}|G(j\omega)H(j\omega)| = -0.769$$

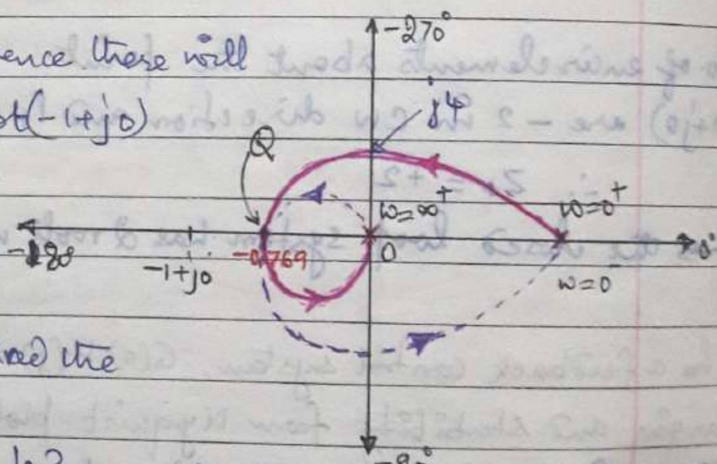
$$\lim_{\omega\to\sqrt{10}}\angle G(j\omega)H(j\omega) =$$

- when $\omega \to \infty$

$$\lim_{\omega\to\infty}|G(j\omega)H(j\omega)| = 0^\circ$$

$$\lim_{\omega\to 0}\angle G(j\omega)H(j\omega) = -27$$

- As the system is type '0' hence there will not be any closing about pt$(-1+j0)$. Only mirror image will be formed.




- $N=0$, $P_+=0$, so $Z_+=0$ and the system is stable

- $G.M = \dfrac{1}{l(OQ)} = \dfrac{1}{0.769} = 1.3$

or $G.M = 20\log_{10}\dfrac{1}{OQ} = 20\log_{10}\dfrac{1}{0.769}$

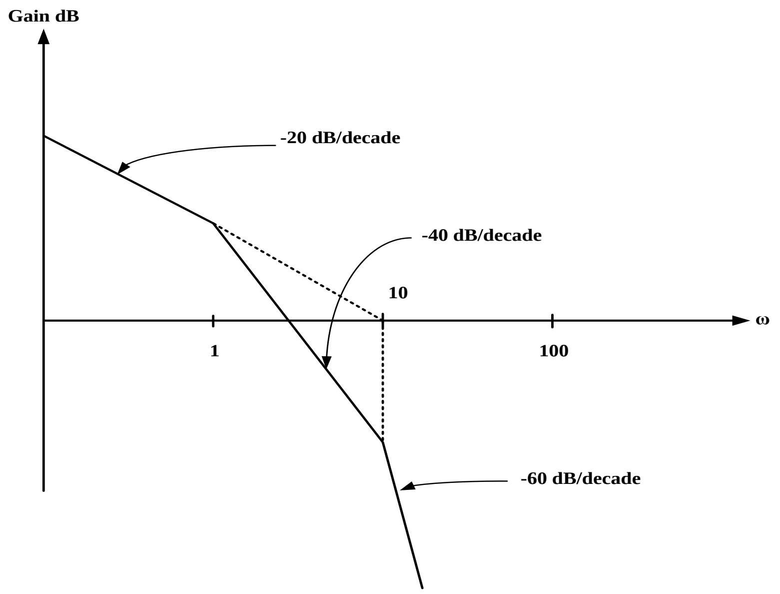
$G.M = 20\log_{10}(1.3)$

$$\boxed{G.M = +2.27\,dB}$$

# LNCT GROUP OF COLLEGES

## Assignment on Unit-IV Frequency Domain Analysis

1. Find the transfer function for the shown Bode plot.




2. A unity feedback system has an open loop transfer function of $G(s) = \frac{K\,e^{-0.5s}}{(s+1)}$. Analytically determine the critical value of K for stability and verify by examining the Nyquist plot.

3. Plot the Bode diagram of the open-loop transfer function $G(s)H(S) = \frac{20(s+1)}{s(s+5)(s^2+2s+10)}$. Determine the gain margin, phase margin, phase-crossover frequency, and gain-crossover frequency.

4. The open loop transfer function of a unity feedback control system is given by $G(s) = \frac{K}{s(s+1)(1+0.1s)}$. Determine phase-crossover frequency, and gain-crossover frequency from the Bode plot. Find the value of K for which the gain crossover frequency is 5 rad/sec.

5. The open loop transfer function of a unity feedback control system is given by $G(s) = \frac{K}{s(0.02s+1)(1+0.04s)}$. If the gain K produces a phase margin of 45°, find K and the corresponding gain margin.

6. Using Nyquist stability criterion comment on the closed loop system stability for a system whose loop transfer function is given by

(i) $G(s)H(S) = \frac{K(s+3)}{s(s-1)}$

(ii) $G(s)H(S) = \frac{100}{(1+0.1s)(1+0.2s)(1+0.3s)}$

(iii) $G(s)H(S) = \frac{(s+5)}{(s+3)(s-1)}$

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5) Automatic Control systems by Benjamin C. Kuo, Wiley India Pvt. Ltd, 9th Edition.